21.8.2020

B. Sc. Part III Diff Eqns (contd) [SOEVC (contd.)] (4^{th} Paper)

Q. Solve $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 3y = 0$, if $y = x$ is a solution of it.

Soln. The given equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 3y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{2x}{1-x^2}\frac{dy}{dx} + \frac{3}{1-x^2}y = 0 \quad \text{---} (1)$$

comparing it with the eqn

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Here, $P = \frac{-2x}{1-x^2}$, $Q = \frac{3}{1-x^2}$, $R = 0$.

Given that x is a soln of (1)

Let $u = x$.

Let the complete soln be $y = uv$.

Then, v is given by

$$\frac{d^2v}{dx^2} + \left(P + \frac{2}{u}\frac{du}{dx}\right)\frac{dv}{dx} = \frac{R}{u}$$

$$\Rightarrow \frac{d^2v}{dx^2} + \left(\frac{-2x}{1-x^2} + \frac{2}{x}\frac{d(x)}{dx}\right)\frac{dv}{dx} = 0$$

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$$\Rightarrow \frac{d^2v}{dx^2} + \left(\frac{-2x}{1-x^2} + \frac{2}{x}\right)\frac{dv}{dx} = 0$$

$$\Rightarrow \frac{d^2v}{dx^2} + \frac{-2x^2+2-2x^2}{x(1-x^2)}\frac{dv}{dx} = 0$$

$$\Rightarrow \frac{d^2v}{dx^2} + \frac{2(1-2x^2)}{x(1-x^2)}\frac{dv}{dx} = 0$$

Put $\frac{dv}{dx} = z \Rightarrow \frac{d^2v}{dx^2} = \frac{dz}{dx}$

$$\Rightarrow \frac{dz}{dx} + \frac{2(1-2x^2)}{x(1-x^2)} z = 0$$

$$\Rightarrow \frac{dz}{z} + \frac{2(1-2x^2)}{x(1-x^2)}\, dx = 0$$

$$\Rightarrow \frac{dz}{z} + 2\left[\frac{(1-x^2)-x^2}{x(1-x^2)}\right]dx = 0$$

$$\Rightarrow \frac{dz}{z} + 2\left[\frac{dx}{x} - \frac{x}{1-x^2}\right]dx = 0$$

$$\Rightarrow \frac{dz}{z} + 2\frac{dx}{x} + \frac{-2x\,dx}{1-x^2} = 0$$ Integrating, we get

$$\Rightarrow \log z + 2\log x + \log(1-x^2) = \log c$$

$$\Rightarrow z x^2(1-x^2) = c$$ But $z = \frac{dv}{dx}$

$$\Rightarrow \frac{dv}{dx} \cdot x^2(1-x^2) = c \Rightarrow dv = \frac{c\,dx}{x^2(1-x^2)}$$

$$\Rightarrow dv = c\,\frac{dx}{x^2(1-x^2)}$$

$$\Rightarrow dv = c\left[\frac{(1-x^2)+x^2}{x^2(1-x^2)}\right]dx$$

$$\Rightarrow dv = c\left[\frac{dx}{x^2}+\frac{dx}{1-x^2}\right]$$

Integrating, we get

$$\Rightarrow v = c\left[-\frac{1}{x}+\frac{1}{2}\log\frac{1+x}{1-x}\right]+K \quad -(2)$$

So, the required soln is

$y = uv$ where $u = x$ and v is

given by (2).